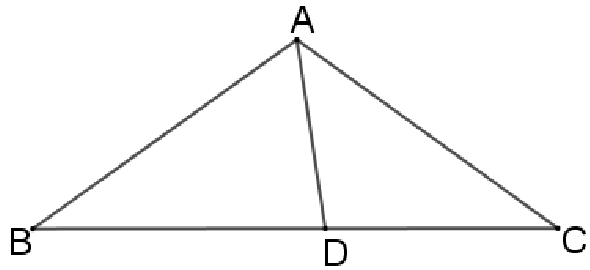
01.08.2024 Cash Award Rider - Author's Solution

Before going in to the solution, let us see an important corollary of Stewart's Theorem.



The above triangle is isosceles with AB=AC. AD is a cevian. Now, let us find out AD^2 **with the help of Stewart's Theorem.**

$[(AB^2 \times DC) + (AC^2 \times BD)] = BC[AD^2 + (BD \times DC)]$

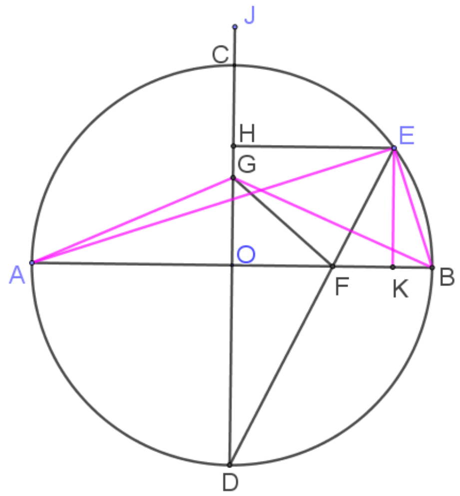
$AB^2(BD + DC) = BC\,[AD^2 + (BD \times DC)]$ **(∵AB=AC)**

$AB^2 \times \cancel{B}C = \cancel{B}C\,[AD^2 + (BD \times DC)]$ **(∵ (BD+DC)=BC)**

$AB^2 = AD^2 + (BD \times DC)$

$AD^2 = AB^2 - (BD \times DC)$ **or**

=$AC^2 - (BD \times DC)$



Let us use this result in our solution.

Now, let us go in to the Solution.

Solution :

Construction :

Join GA, GB, AE & BE. Draw EK ⊥ AB.

$\widehat{AD} = \widehat{BD}$ **(1/4 of the circumference)**

$\Longrightarrow \angle AED = \angle BED$ **[equal arcs have equal angles]**

$\Longrightarrow$ EF **is the angle bisector of** $\angle AEB$.

$\Longrightarrow\ EF^2 = (AE \times EB) - (AF \times FB)$ **---------------------- (1)**

GAB is an isosceles triangle and GF is a cevian. (∵GO is the perpendicular bisector of AB)

As per the above result under Stewart's Theorem,

$GF^2 = GB^2 - (AF \times FB)$ **------------------(2)**

EF = GF (given) -----------------------------------(3)

(1), (2) & (3) →

GB^2 **= AE ×** EB **------------------------------------(4)**

$AE \times EB = \mathrm{AB} \times EK$ **--------------(5) (twice the area of the** ΔAEB)

(4) & (5) $\rightarrow$

$GB^2 = EK \times AB = OH \times AB = OH \times 2OB$

$\Longrightarrow OG^2 + OB^2 = \text{OH} \times 2OB$

$\Longrightarrow OG^2 = (OH \times 2OB) - OB^2$

= OB (2OH − OB)

= OC (2OH − OC)

= OC (OJ − OC)

$OG^2 = OC \times CJ$ ----------------------- Proved

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